

## The Stationarity of Estimated Beta of Individual Securities and Portfolios

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### I. Introduction

In the Capital Asset pricing Model (CAPM) developed by Sharpe [22], Lintner [14] and Mossin [6] the equilibrium rate of return of a security is related to its systematic risk (the beta coefficient) in the following form:

$$E(R_i) = R_{it} + \beta_i [E(R_{mt}) - R_{it}] \quad (1)$$

where  $E(R_i)$  and  $E(R_{mt})$  are the expected return on the security and market portfolio, respectively and  $R_{it}$  is the risk - free rate return. Application of the CAPM to the cost of capital, capital budgeting and portfolio management requires an estimate of the true value of beta.

The most common procedure for estimating  $\beta$  has been Sharpe's [21] market model. The estimated  $\beta$  from the market

model will be biased and inefficient unless  $\beta$  is stationary. The CAPM by itself does not imply either beta stationarity or any particular form of nonstationarity. Market participants are presumed to know the beta coefficients of the available securities.

Previous research on the stationarity of  $\beta$  has yielded mixed conclusions. Blume [5] and Levy [13] concluded that the  $\beta$  of individual securities is nonstationary; but as the numbers of securities in a portfolio increases, the  $\beta$  of the portfolio becomes more stationary. Porter and Ezzell [18] and Tole [25] found that by altering the portfolio selection procedure the stationarity of portfolio  $\beta$ 's observed by Blume and Levy was reduced considerably. Fabozzi and Francis [10] concluded that neither alpha nor beta is significantly different between bear and bull markets.

A common shortcoming in previous studies is that the question of "how to measure the stationarity of  $\beta$  of a single asset" was not addressed. The correlation analysis used in the previous studies provides only an overall measure of stationarity for all the assets and can not be used to study the stationarity of  $\beta$  for a single asset.

The purpose of this study is to analyze the shortcomings of the statistical techniques used in the previous studies and to re-examine the stationarity of  $\beta$  using more appropriate and more powerful statistical techniques than have been employed in the previous studies. In section two previous research on the

stationarity of  $\beta$  and their limitations are analyzed.

Section three explains the sample and introduces two econometric techniques which can be used to test the stationarity of  $\beta$  of individual securities and portfolios. Empirical results are discussed in sections four and five. The conclusions and implications are given in section six.

## II- Background

### A. Previous Studies

Blume [5] was the first researcher who addressed the question of the stationarity of  $\beta$ . His basic procedure was first to form six non-overlapping 84 - month time periods from July 1926 through June 1968. For each 84-month subperiod,  $\beta$  was estimated from the market model as:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \quad (2)$$

Where  $R_{it}$  and  $R_{mt}$  are the returns on security  $i$  and the market portfolio in month  $t$ . Blume assessed the stationarity of  $\beta$  by computing the product moment and rank order correlation coefficients of  $\beta$ 's in different subperiods for portfolios of size 1,2,4,7,10,20,35,50 and 100. Portfolios were formed by first ranking the security  $\beta$ 's in period  $t$ . Portfolios of size  $N$  then were formed by selecting the first  $N$  securities for portfolio one, the second  $N$  securities for portfolio two, and so forth. In period  $t+1$  portfolio one had the same securities as in period  $t$ . The same conditions held for all of the other portfolios. The degree of

stationarity in  $\beta$  was measured by the magnitude of the correlation coefficients between the  $\beta$ s of portfolios in two periods. His results showed that the Correlation coefficient increased as the number of securities in a portfolio become larger. Based on the foregoing results he concluded that  $\beta$ 's of portfolios are more stationary than  $\beta$ 's for individual securities.

Levy [13] tested the stationarity of  $\beta$  in a manner very similar to Blume. His time period was shorter and weekly returns were used. However, the essence of his methodology for forming portfolios was the same as Blume's. He also used the product moment and rank order correlation coefficients to measure the stationarity of  $\beta$ . His conclusions were that the  $\beta$ 's are remarkably stationary for large portfolios and less stationary as the portfolio size declines.

Porter and Ezzell [18] replicated Blume's study and added a random portfolio selection comparison. That is, in addition to forming portfolios on the basis of ranked security  $\beta$ 's Porter and Ezzell formed the same size portfolios with randomly selected securities. The purpose of random selection was to remove the inherent numerical separation that results from forming portfolios on the basis of ranked security  $\beta$ 's. Their research indicates that if random selection is used to form portfolios, the stationarity of  $\beta$  shows no discernible relationship with the number of securities in a portfolio. Likewise, the average correlation coefficient is not very high-about 0.666. Their conclusion is that the stationarity of

$\beta$  is a function of the portfolio selection process.

Tole [25] argued that to study the stationarity of  $\beta$  in a realistic environment portfolios should be formed in a manner similar to that used, by portfolio managers. He used portfolios recommended by brokerage firms, research services, and financial periodicals. The remainder of his study basically followed Blume's methodology. The results of Tole's research were that the  $\beta$ 's of portfolios selected on the basis of technical and fundamental factors are less stationary than the  $\beta$ 's of randomly selected portfolios.

A study by Sharpe and Cooper [23] evaluated the stationarity of  $\beta$  for risk - return classes. Their procedure was to calculate the  $\beta$  of the securities, rank the securities by  $\beta$ , and separate the securities into ten risk-classes where each risk - class is a decile. The foregoing procedure was repeated for six years. Stationarity was measured by constructing a transition matrix. The entries in the transition matrix were the proportion of the securities that were in risk-class  $i$  in period  $t$  and were in risk-class  $j$  in period  $t+k$ . No tests for statistical significance were done. Their conclusion is that securities possess substantial stability over time.

Fabozzi and Francis [10] used dummy variables to examine the stationarity of alpha and beta for individual securities over bear and bull markets. Their study varied significantly from previous research in that the significance tests were applied to individual securities in contrast to one aggregate test for stationarity on the

entire sample of securities. Fabozzi and Francis' conclusion is that the varying economic forces associated with bear and bull markets do not result in significantly different betas for the different market conditions.

### B. Limitations of Previous Studies

Previous studies on the stationarity of  $\beta$  contain two major problems. The first problem is that all of the previous authors used equation (2), or some minor modification of it, to estimate  $\beta$ . In doing so they implicitly assumed that during the estimation period  $\beta$  is stationary. For example, Blume used seven years of data to estimate  $\beta$ . Therefore, Blume's analysis assumes that  $\beta$  is stationary within each of the seven-year time periods studied. Fabozzi and Francis [10] assumed that  $\beta$  is stationary during a bear or bull market. A more appropriate approach is to use a time series analysis that allows  $\beta$  to change during the estimation period and then evaluate the stationarity of  $\beta$  within the specified time frame.

Another limitation of the correlation coefficient (P) is that it can not be used to determine the stationarity of  $\beta$  for a single asset. That is, if  $p < 1$ , we can not conclude that all of the individual assets have nonstationary  $\beta$ 's, or which individual assets have nonstationary  $\beta$ 's or even what proportion of the assets have nonstationary  $\beta$ 's. Since in many applications, for example estimating The cost of equity for public utilities, it is necessary to

determine the stationarity of  $\beta$  for a single asset, techniques other than the correlation coefficient must be used.

An ideal test for stationarity would evaluate the constancy of the individual asset beta over time by examining whether the regression coefficients in the market model vary over time. In the following methodology section we describe the data for our study, the procedure used to form portfolios, and two econometric procedures for evaluating the constancy of the regression relationship.

### **III. Methodology**

#### **A. Data**

The universe of assets considered consisted of all securities listed on the Center for Research in Securities Prices (CRSP) tape with no missing data for the period of January, 1957, through December, 1976. A random sample, without replacement, of 200 securities was selected to study. Monthly security returns included both price changes and cash dividends. Fisher's Value Weighted Market Index was used as a proxy for the market portfolio.

Since we wanted to insure that our results were not time period specific, we separated the twenty year time span into four non-overlapping five year subperiods. A five year subperiod was selected because this is a common estimation period for  $\beta$ . The establishment of subperiods also allowed us to apply the product moment and rank correlation procedures as a comparison to the

econometric procedures.

### B. Portfolio Construction

Portfolios consisting of 1,2,5,10 and 20 securities were formed in two ways. For each portfolio formation method, equation (1) was used to estimate  $\beta$  for each security in each subperiod. In portfolio method one securities first were ranked by  $\beta$ . Then portfolios of size N were formed by placing the first N securities in portfolio one, the second N securities in portfolio two, and so forth. The number of portfolios of each size was  $200/N$ .

The second set of portfolios was formed by randomly selection 50 portfolios of size N ( $N=1,2,5,10$  and  $20$ ) in each subperiod. Random portfolios were used to determine if porter and Ezzell's [18] conclusion that the stationarity of  $\beta$  for portfolios is a function of the portfolio selection process remains valid when test procedures other than correlation coefficients are used to measure stationarity.

### C. Statistical Tests

An appropriate statistical test for the stationarity of  $\beta$  should identify departures from constancy of  $\beta$  over time. Therefore, to correctly study the behavior of  $\beta$  over time, equation (2) is rewritten as:

$$R_t = \alpha_t + \beta_t R_{mt} + \varepsilon_t \quad (3)$$

Where the subscript t on  $\alpha$  and  $\beta$  indicates that  $\alpha$  and  $\beta$  may



vary over time. In matrix notation equation (3) becomes:

$$Y_t = X_t' \beta_t + \varepsilon_t \quad t = 1, 2, \dots, T \quad (4)$$

Where  $\beta_t' = (\alpha_t \beta_t)$ ,  $X_t$  is a  $(2 \times T)$  matrix of observations on the explanatory variables  $(1 R_{mt})$ ,  $Y_t$  is the vector of returns on an individual security or portfolio, and  $\varepsilon_t$  is a vector of disturbances.

The null hypothesis for stationarity was formulated as:

$$H_0: \beta_1 = \beta_2 = \dots = \beta_T \quad t = 1, 2, \dots, T.$$

The alternate hypothesis was that not all of the  $\beta$ 's were equal. It is assumed that the variance of error term is constant in each time period.

The stationarity problem is a special case of a general class of problems concerned with the detection of changes in model structure over time. Early works on detecting the changes in model structures were done by Anscombe [1] and Anscombe and Tukey [2]. They used ordinary least square (OLS) residuals to investigate departures from model specification. A major problem with this procedure is that the plot of the OLS residuals, or the plot of their squares, against time is not a very sensitive indicator of small or gradual changes in the  $\beta$ 's.

To improve the effectiveness of the OLS residual analysis in detecting structural changes in the regression model, Page [17], Barnard [3], and Woodward and Goldsmith [26] suggested that the OLS residuals be replaced by the cumulative sum (cusum) of the OLS residuals. That is, the plot of the OLS residuals ( $e_t$ ) should be replaced with a plot of the scaled cusum of OLS

residuals  $Z_r$  where:

$$Z_r = \sum_{t=1}^r e_t / \hat{\sigma} \quad r = 1, 2, \dots, T.$$

Dividing the cusum of OLS residuals by  $\hat{\sigma}$ , the estimated standard deviation of the OLS residual, eliminates the irrelevant scale factor. The difficulty with this approach is that there is no known method of assessing the significance of the departure of the plot of  $z_r$  from its expected value line  $E(Z_r) = 0$ . The foregoing holds because, as shown by Mehr and McFadden [15], the covariance function  $E[Z_r, Z_s]$  does not reduce to a manageable form.

To avoid the problems associated with the cusum of OLS residuals, Brown and Durbin [17] and Brown, Durlin, and Evans (BDE) [8] proposed using recursive residuals. BDE have shown that under the null hypothesis of stationarity the recursive residuals are uncorrelated with zero mean and constant variance and therefore are independent under the normality assumption. Recursive residuals are preferable to OLS residuals for detecting changes in  $\beta$  because until a change takes place the recursive residuals behave exactly as specified by the null hypothesis.

Recursive residuals are defined as:

$$w_r = (y_r - x_r' b_{r-1}) / [1 + x_r' (X_{r-1}' X_{r-1})^{-1} x_r]^{1/2} \quad r = k+1, \dots, T \quad (5)$$

Where  $k$  is the number of regression coefficients,  $X_{r-1}' = [x_1, \dots, x_{r-1}]$ ,  $b_r = (X_r' X_r)^{-1} X_r' Y_r$ , and  $Y_r = (y_1, \dots, y_r)$ . The numerator of equation (5) may be interpreted as a one period prediction error.

BDE used the recursive residuals to construct the cusum of

recursive residuals and cusum of squares tests. Garbade's [11] simulation study indicates that the cusum of squares test is more powerful than the cusum test. Garbade also indicated that the cusum of squares test never gave misleading results when the coefficients were in fact stationary. Also, the cusum of squares test detects nonstationarity in  $\beta$  even if  $\beta$  changes randomly over time. For these reasons, we used the cusum of squares test to test for stationarity of  $\beta$ .

The cusum of squares test uses the cumulative sum of the squares of the recursive residuals:

$$s_r = \sum_{j=k+1}^r w_j^2 / \sum_{j=k+1}^T w_j^2, \quad r = k + 1, \dots, t. \quad (6)$$

Where  $s_r$  is a monotonically increasing sequence of positive numbers with  $S_1 = 1$ . Under the null hypothesis of stationarity,  $S_t$  follows a beta distribution with a mean of  $(r-k) / (T-K)$ .  $H_0$  is rejected if  $|S_t - ((r-k)/(T-k))| > C_\alpha$  for any  $r \in [k+1, T]$ . The value of  $C_\alpha$  for the desired confidence level and sample size is given in Table 1 in Durbin [9]. The result of this test will indicate whether or not  $\beta$  is stationary.

Another method for testing the stationarity of  $\beta$  is to consider  $\beta$  as a linear function of time. That is, for some economic or behavioral reasons,  $\beta$  either increases or decreases over time. Specification of  $\beta$  as a function of time is consistent with Blume's [6] explanation of the regression tendency of  $\beta$ . The foregoing hypothesis can be tested by a general linear test. Under the null

hypothesis of stationarity, equation (4), can be stated as:

$$Y_t = X_t' \beta_0 + \varepsilon_t \quad (7)$$

Where  $\beta_0$  denotes that the regression coefficients of the market model are stationary. Equation (7) usually is called a reduced model. Under the alternative hypothesis the regression coefficients are assumed to change as a linear function of time, or

$$y_t = x_t' \beta_t + \varepsilon_t \quad (8)$$

where:

$$\beta_t = \beta_0 + \delta_{1t} \quad (9)$$

Substitution of (9) into (8) yields

$$Y_t = X_t' (\beta_0 + \delta_{1t}) + \varepsilon_t$$

which usually is called the full model. The null hypothesis of stationarity now can be tested by a comparison of the mean - square increase in the explained variation with the error variance.

The test statistic is calculated as:

$$F = \frac{SSE(R) - SSE(F)}{df(R) - df(F)} \div \frac{SSE(F)}{df(F)}$$

where SSE(R) and SSE(F) are the error sum of squares of the reduced and full models, respectively. Likewise, df(R) and df(F) are the degrees of freedom associated with SSE(R) and SSE(F), respectively.

#### IV. Empirical Results Using Ranked Portfolios

##### A. Correlation Coefficients

The standard of comparison for stationarity of  $\beta$  studies has been the research by Blume [5]. The correlation coefficients in

our study (Table I), both product moment and Spearman's rank order, tend to be smaller than what Blume found. However, the general tendency of the correlation coefficients to converge to one as the number of securities in a portfolio increases is in agreement with Blume's results. The differences in magnitudes may be due to differences in the time periods studied, the number of portfolios of each size used, or the sample studied. The limitations associated with the correlation coefficients makes it difficult to draw firm conclusion concerning the stationarity of  $\beta$ .

Portfolio Size	Time Period					
	1/57-12/61 and 1/62-12/66		1/62-12/66 and 1/67-12/71		1/67-12/71 and 1/72-12/76	
	P.M.	Rank Order	P.M.	Rank Order	P.M.	Rank Order
1	.475	.439	.479	.439	.489	.465
2	.577	.540	.582	.537	.596	.557
5	.726	.681	.707	.714	.758	.683
10	.788	.725	.851	.807	.787	.737
20	.902	.927	.900	.891	.8792	.855

Table I- Product Moment And Rank Order Correlation Coefficients Of betas For Ranked Portfolios of N Securities

### B. Econometric Tests

Table II lists the percent of the portfolios of size N with significantly nonstationary  $\beta$  as measured by the cusum of squares and the time trending regression tests. The significance level was

five percent. For the cusum of squares test the percent of portfolio that were significantly nonstationary generally declined as

Portfolio Size	Cusum of Squares	Time Tredning
January 1957- December 1961		
1	31.0	6.5
2	20.0	10.0
5	12.5	15.0
10	10.0	10.0
20	10.0	20.0
January 1962 - December 1966		
1	36.0	13.0
2	40.0	12.0
5	35.0	12.5
10	20.0	15.0
20	20.0	20.0
January 1967 - December 1971		
1	10.0	7.0
2	13.0	6.0
5	10.0	12.5
10	10.0	20.0
20	0.0	10.0
January 1972- December 1976		
1	26.0	8.0
2	25.0	8.0
5	12.5	0.0
10	15.0	0.0
20	10.0	0.0

Table II - Percent of Ranked Portfolio With Significantly Nonstationary  $\beta$

the number of securities in a portfolio increased. This result is consistent with the correlation coefficient studies. In addition, the cusum of squares test indicates that only about one-third of the individual securities had nonstationary  $\beta$ 's or the majority of the securities had stationary  $\beta$ 's. Thus the conclusion of previous researchers, based upon correlation coefficients, that  $\beta$ 's for individual securities are highly unstable seems to be unwarranted. Furthermore, the decline in the percent of portfolio with nonstationary  $\beta$  as  $N$  increase implies that the variation in  $\beta$  of individual securities is primarily random and the random variations in  $\beta$  cancel each other out as the number of securities in a portfolio increases.

The time trending test detects if  $\beta$  is a linear function of time. In other words, it test if  $\beta$  is increasing or decreasing over the estimation period. The results of this study indicates that the liner time trending tests generally identified more portfolios as nonstationary than expected by chance but fewer portfolios as nonstationary that the cusum of squares tests. One reason for this result is that the time trending test detects only a special type of nonstationarity (systematic change). Whereas the cusum of squares test detects both systematic and stochastic nonstationarity in  $\beta$ .

The results of the time trending test also indicated that the majority of the nonstationary portfolios were those which had either extremely low or high betas in the portfolio formation

period. This result is consistent with Blume's conclusion with regard to the regression tendencies of  $\beta$ 's.

In other words, the beta of a low (high) risk portfolio tends to become larger (smaller) over the estimation period. In contrast to the cusum of squares tests the time trending tests indicated, with the exception of 1972-76 time period, that the percent of nonstationary portfolios did not decline as the portfolio size increased. An intuitive explanation of this result is that the portfolios were formed from ranked values of  $\beta$ 's. In the ranked portfolio selection, the extremely low or high beta portfolios contain securities with extremely low or high betas and since betas of these securities tend to move in the same direction, then betas of these portfolios also tend to move in the same direction as betas of the securities. As a result, the percent of nonstationary portfolios did not decline as the portfolio size increased.

#### V. Empirical Results Using Random Portfolios

The results of forming random portfolios are reported in Table III. A comparison of Tables II and III indicates that the results of the tests on the ranked and random portfolios are in the same general direction but the ranked portfolio respond more systematically to changes in the portfolio size. The time trending tests indicated that the percent of nonstationary portfolios were smaller than those in the ranked portfolios. An obvious explanation of this result is that in the random selection method



Portfolio Size	Cusum of Squares	Time Tredning
	January 1957- December 1961	
1	18.0	4.0
2	24.0	6.0
5	16.0	10.0
10	12.0	4.0
20	6.0	10.0
	January 1962- December 1966	
1	32.0	12.0
2	34.0	12.0
5	22.0	16.0
10	18.0	12.0
20	18.0	16.0
	January 1967- December 1971	
1	22.0	2.0
2	14.0	16.0
5	8.0	10.0
10	14.0	16.0
20	6.0	24.0
	January 1972- December 1976	
1	28.0	2.0
2	26.0	12.0
5	32.0	12.0
10	28.0	2.0
20	26.0	4.0

Table III - Percent of Random Portfolios With Significantly Nonstationary b

each security has an equal probability of being included in a portfolio. Thus the random selection method substantially reduces the probability of obtaining an extremely low or high beta portfolio. As a result there is less chance of observing the regression tendencies in the betas of random portfolios.

These results suggest that the portfolio selection method affects the outcomes of the stationarity tests. Therefore, our study supports Porter and Ezzell's conclusion that the stationarity of beta is a function of the portfolio selection method.

## VI. Conclusions

The results of this study agree with those of previous researchers who concluded that the stationarity of beta increases as the number of securities in a portfolio increases. However, for the four time periods studied, the maximum percentage of individual securities with nonstationary  $\beta$ 's was 36.0%. Therefore, in contrast to previous studies which concluded that  $\beta$ 's of individual securities were highly unstable we conclude that the majority of individual security  $\beta$ 's are stationary. This result is significant in that many applications of  $\beta$  require an estimate of individual security  $\beta$ 's and one of the major criticisms of  $\beta$  has been that  $\beta$  is nonstationary. This criticism appears to have been overstated as a result of an inappropriate application and interpretation of the correlation coefficient as a measure of stationarity.

The time trending tests showed that betas of the extremely low (high) risk portfolios become larger (smaller) over time. This result supported the regression tendencies of betas which was reported by Blume [6]. We also found that the regression tendencies of betas occur more often in the ranked portfolio selection method than in the random portfolio selection method. This result is consistent with the findings of porter and Ezzell [18] who stated the portfolio selection method affects the outcomes of the stationarity tests.

The results of this study suggest that before using the simple market model the stationarity of beta should be tested by the susum of squares test, time trending regression or other techniques wich are designed to test the stationarity of regression coefficients [8]. If beta is found to be nonstationary, then the market model should be replaced by a random coefficient regression model [19 and 24] or a systematic parameter variation model [4 and 20], depending on whether beta is changing randomly or systematically over time.

Although we found that the majority of individual securities and portfolios have a stationary beta, the application of beta could be improved if several other questions could be answered. Some of these are:

- (1) what is the best historical time period (60 months, 48 months, etc.) for predicting beta?
- (2) Does the application of data which is more closely

associated with actual trading activity - for example, daily observations influence the observed stationarity of beta and the prediction of beta?

(3) Can the point in time when beta switches from being stationary to nonstationary be identified and predicted?

(4) What factors are responsible for the nonstationarity of beta and how can these factors be incorporated into the estimation procedure of beta?.

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